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Statistical Models for Estimating Optimum Row Width

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Summary

Two relations between crop yield and row width have been derived from the existing relations of plant geometry. One of these two relations belongs to the class of quadratic inverse polynomials and the other to that of second degree ordinary polynomial without a constant term. The two relations were fitted to the two sets of experimental data for sorghum and groundnut, under dryland conditions. These relations are useful in identifying the optimum inter-row spacing for obtaining maximum yield.
Keywords : Inverse polynomials, weighted least-squares, inter-row distance, optimum row width.

Introduction

Plant population and row spacing are important in crop production, particularly under dryland conditions. A deficiency of soil water limits production in dry environment. To obtain the maximum yield, the plant population and planting pattern must optimise the use of available water in the soil. There is a long history of investigation of relationships between plant population and crop yield. Willey and Heath [6] reviewed these relationships. Under moisture stress conditions, which are very common in dry environment, use of a suitable inter-row is important as water use by plants and loss in moisture due to evaportranpiration depend on inter-row spacing.

Studying the response in crop yield due to inter and intra-row spacings simultaneously provides information about the optimum plant geometry. Goodall [3] and Berry [1] suggested different types of relations for this situation. For some crops and in some situations there exists a wide plateau in yield over a range of plant densities. In some such situations, crop yield, depends greatly on row spacing. Some experiments were conducted at the Co-operating Centres of the All India Co-ordinated Research Project for Dryland Agriculture (AICRPDA) to study the effect of row spacing alone or in combination with other management factors such as fertilisers, cultivars, etc., at a constant plant density or seed rate. From the existing relations of crop geometry suggested by Goodall and Berry, the relations between crop yield and inter-row spacing have been deduced.

2. *Model*

The relation suggested by Goodall [3] is

$$w = a\ x_1^{b_1}\, x_2^{b_2} \tag{1}$$

and by Berry [1] is

$$\frac{1}{w^{\theta}} = a + \frac{b_1}{x_1} + \frac{b_2}{x_2} + \frac{c}{x_1\, x_2} \tag{2}$$

where w is the yield per plant, x_1 and x_2 are inter-row spacings and θ, b_1, b_2 and c are constants. Equation (2) is an extension of the Bleasdale and Nelder [2] yield-density relation modified to include a term for plant rectangularity. Equation (1) can be written in terms of the variables representing rectangularity and plant density as

$$w = a \left(\frac{x_1}{x_2}\right)^{\frac{b_1 - b_2}{2}} \left(x_1 x_2\right)^{\frac{b_1 + b_2}{2}} \tag{3}$$

where $\left(\frac{x_1}{x_2}\right)^{1/2}$ represents rectangularity and $\left(x_1\, x_2\right)^{-1}$ represents plant density. If the plant density is constant, $x_1 x_2 = k$, where k is a constant and (3) reduces to

$$y = a_1 \left(\frac{x_1}{x_2}\right)^{c/2} \tag{4}$$

where y the yield per unit area, is equal to p w, where $p = 1/(x_1 x_2)$ is the plant density, $a_1 = a\,(x_1\, x_2)^{[(b_1,\, b_2)/2]^{-1}}$ and $c = (b_1 - b_2)$.

In equation (4) the effect of rectangularity on crop yield is significant when c is significantly different from zero. However, this equation does not explain decrease in yield due to increase in row width. Hence it is extended by incorporating one more term as,

$$y = a\left(\frac{x_1}{x_2}\right)^{c/2} + b\left(\frac{x_1}{x_2}\right)^{c} = \alpha_g x_1^c + \beta_g x_1^{2c} \tag{5}$$

where $\alpha_g = a\left(x_1 x_2\right)^{-c/2}$, $\beta_g = a\left(x_1 x_2\right)^{-c}$. This is generalized version of the ordinary second degree polynomial with no constant term. When c=1 it is a second degree polynomial with no constant.

With $x_1x_2 = k$, equation (2) reduces to

$$\frac{x_1}{w^\theta} = b_1 + a_1 x_1 + b_2' x_1^2 \tag{6}$$

where $a_1 = (a + c/k)$, $b_2' = b_2/k$ and $x_1x_2 = k$. Since y is the product of w and p, the form of the curve is not changed, by replacing w by y, since p is a constant here. So we have,

$$\frac{x_1}{y^\theta} = \alpha_b + \beta_b x_1 + \gamma_b x_1^2 \tag{7}$$

where $\alpha_b = b_1\left(\frac{1}{x_1x_2}\right)^{-\theta}$, $\beta_b = a_1\left(\frac{1}{x_1x_2}\right)^{-\theta}$ and $\gamma_b = b_2'\left(\frac{1}{x_1x_2}\right)^{-\theta}$

The optimum row width (X_{1opt}) to get the maximum yield per unit area is given by

$$X_{1opt} = \left(\frac{\alpha_b}{\gamma_b}\right)^{1/2}$$

which is independent of θ. The objective in these studies is to find X_{1opt}, and hence θ is taken as unity. The final equation is,

$$\frac{x_1}{y} = \alpha_b + \beta_b x_1 + \gamma_b x_1^2 \tag{8}$$

The equations (5) and (8) were fitted to two sets of experimental data.

3. *Fitting to Experimental Data*

The data from two experiments conducted at two locations of AICRPDA, viz., Kovilpatti and Rajkot were used. Five row widths, viz., 30, 45, 60, 75 and 90 cm were tested in four randomized blocks and the data are given in Table 1. The test crops were post-monsoon sorghum at Kovilpatti and groundnut and Rajkot.

Table 1. Yield of Post-Monsoon Sorghum and Groundnut for Different Inter-row Spacings at Constant Plant Density

Inter-row spacing (cm)	Sorghum				Groundnut pod yield (t/ha) Replication			
	1	2	3	4	1	2	3	4
30	2.833	2.747	2.513	2.193	0.417	0.434	0.417	0.243
45	3.636	3.745	4.115	3.788	0.425	0.451	0.451	0.347
60	2.747	2.817	2.740	2.725	0.464	0.556	0.495	0.408
75	2.500	2.703	2.695	2.703	0.433	0.417	0.442	0.370
90	1.855	1.890	1.838	1.883	0.392	0.417	0.391	0.373

At Kovilpatti the plant density was kept constant at 148148 plants/ha and at Rajkot a seed rate of 80 kg/ha was kept constant. Equation (7) is similar to the Bleasdale – Nelder equation with an extra term p^2 which belong to the general class of inverse polynomials (Nelder, [5]). To fit this equation, the weighted least squares procedure discussed by Nelder was adopted. the expectation and variance of yield corresponding to ith inter-row distance x_{ij} are

$$E(y_i) = \left(\frac{\alpha_b}{x_{1i}} + \beta_b + \gamma_b x_{1i}\right), \qquad i = 1, 2, \ldots, n$$

$$= \alpha_b z_{1i} + \beta_b z_{2i} + \gamma_b z_{3i}$$

$$V(y_i) = \sigma^2 [E(y_i)]^2, \quad Cov(y_i, y_j) = 0 \quad \text{for} \quad i \neq j$$

where $Z_{1i} = \frac{1}{x_{1i}}$, $Z_{2i} = 1$ and $Z_{3i} = X_{1i}$

The weighted least-squares criterion involves in minimizing the quantity $\sum_i \left[\frac{Y_i}{E(y_i)} - 1\right]^2$

This leads to the normal equations

$$A\,\beta = U,$$

where $A = (a_{pq})$; p, q = 1, 2, 3 ; $a_{pq} = \sum_j Z_{pj} Z_{qj} y_j^2$

$\beta = (\alpha_b, \beta_b, \gamma_b)'$ $U = (u_1, u_2, u_3)'$

and $U_p = \sum_j Z_{pj} y_j$

The residual sum of squares is given by

$$n - \beta' U$$

In fitting relation (5) the parameters were estimated by the ordinary least squares procedure by assuming $V(y_i) = \sigma^2$ for all i and $Cov(y_i, y_j) = 0$ for $i \neq j = 0$. The constant c has to be estimated through iteration. The expected value of the residual sum of squares corresponding to model (5) is $(n-2)\sigma^2$ and for model (7) is $(n-3)\,\theta^2\,\sigma^2$

The estimates of error variances from the replicated data are 0.0299 and 0.0011 for sorghum and groundnut, respectively, and the treatment differences were significant at the 5% probability level. The equations (5) and (8) were fitted to both the sets of data. The adequacy of fit of these models was tested by testing the unexplained residual variation by fitting these models against the estimate of error variance obtained from replicated data. In fitting the equation (5) for sorghum data there was no reduction in RMS when $C \neq 1$, whereas the RMS was minimum when in the neighbourhood of c=0.8 for groundnut data.

The values of the RMS after fitting the ordinary polymial (5) and inverse polynomial (8) with linear and quadratic terms are given in Table-2. Comparison of the RMS with experimental effor shows fitting the inverse quadratic polynomial to the sorghum data and the ordinary quadratic polynomial to the groundnut data adequately explain the variation due to the different row spacings.

Table 2. Residual Spacings Mean Square (RMS) After Fitting Ordinary and Inverse Polynomials Given in (5) and (7) and Error Variances Obtained from the Replicated Data of Sorghum and Groundnut

Crop	Description	df	RMS
Sorghum	Ordinary linear polynomial (through origin)	4	9.0800
	Ordinary quadratic polynomial (through origin)	3	0.8600
	Inverse linear polynomial	3	0.3016
	Inverse quadratic polynomial	4	0.0570
	Experimental error	12	0.0299
Groundnut	Ordinary linear polynomial (through origin)	4	0.0980
	Ordinary quadratic polynomial (through origin with c=0.8)	3	0.0020
	Inverse linear polynomial	3	0.0398
	Inverse quadratic polynomial	4	0.0166
	Experimental error	12	0.0011

The fitted equations are,

Sorghum

$$\frac{x_1}{y} = 25.458 - 0.8176x_1 + 0.01180x_1^2, \quad \hat{\sigma}^2 = 0.0338 \tag{8}$$

Groundnut

$$y = 0.0344x_1^{0.8} - 0.00065x_1^{1.6}, \quad \hat{\sigma}^2 = 0.0013 \tag{9}$$

The estimate of $\hat{\sigma}^2$ are the pooled estimate of RMS and experimental error.

4. *Conclusion*

Either ordinary or inverse polynomial relations appear to be suitable, depending upon the test crop the test crop may be the location and season, for explaining the yield variations due to inter-row spacing. Extrapolation, particularly of ordinary polynomials, needs to be avoided. The optimum row widths obtained from equations (7) and (8) are 46.45 and 60.02 cm, for sorghum and groundnut, respectively. In situation where different treatments such as plant densities of cultivars are tested in different row arrangements, fitting the adequate number of parameters including all these treatments has to be judged by invariance fits, as discussed by Mead [4].

In connection with fitting inverse polynomials, Nelder [5] also discussed the method of maximum likelihood estimation based on the assumption that log (y_1) is normally distributed with Var $(\log y_1)$ $= \sigma^2$. When σ^2 is small, as in the examples, the method of maximum likelihood is a good approximation to weighted least squares. In a simultaneous study of row widths and other management factors such as fertilizer levels, the model (8) can be extended by including these factors, with inverse polynomials. In Table 2 the RMS values are given for linear and as well as quadratic models. the reduction in RMS due to the inclusion of the quadratic term justifies the inclusion in the model.

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